

# 22<sup>ND</sup> INTERNATIONAL WORKSHOP ON LASER RANGING

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RECONNECTING THE ILRS COMMUNITY



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# Thermal Thrust Perturbations, Spin evolution and the Long-Term behavior of LAGEOS II Semi-Major Axis



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# Preamble

Our research activities concerning the development of **perturbation models** and the **precise orbit determination (POD)** of passive geodetic satellites are part of the current experiment **Satellite Tests of Relativistic Gravity (SaToR-G)**. This is an experiment in Fundamental Physics of the **National Scientific Committee 2 (CSN2)** of the Italian **National Institute for Nuclear Physics (INFN)**. See tomorrow *Session 5 @12:45: Fundamental Physics results in testing Gravitation with Laser-Ranged satellites: the LARASE and SaToR-G experiments.*

**SaToR-G** builds on the improved dynamical model of the two **LAGEOS** and **LARES** satellites achieved within the previous project **LARASE (LAsER RAnged Satellite Experiment)**.

The improvements concern the modeling of both **gravitational** and **non-gravitational perturbations (NGPs)**.

In this presentation we focus on the **NGPs** and, specifically, on the **thermal thrust** effects and on the **spin evolution** of the **LAGEOS II** satellite.

# Summary

- POD and long-term behavior of LAGEOS II semi-major axis
- Spin evolution of LAGEOS satellites
- Thermal thrust effects on LAGEOS satellites
- On the decay and rise of the semi-major axis of LAGEOS II
- Conclusions.

# POD and long-term behavior of LAGEOS II semi-major axis



Preliminary POD of LAGEOS and LAGEOS II on a time span of about 28 years

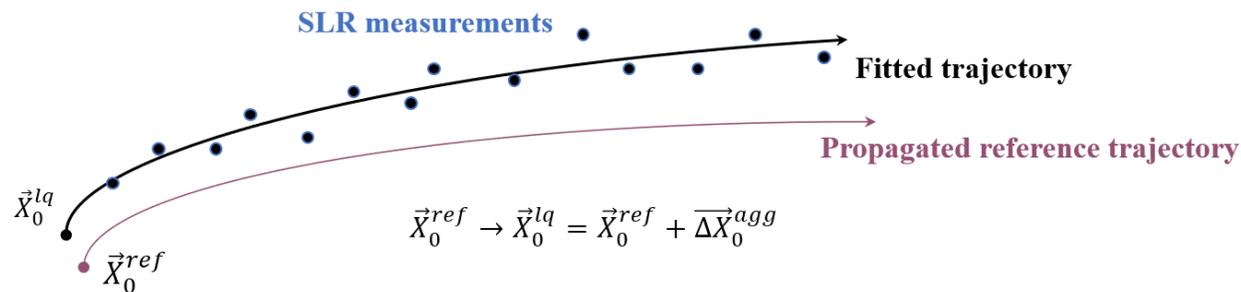
- **GEODYN II s/w**

- Arc length, 7 days
- GR and thermal thrust effects: not modeled
- Empirical accelerations, CR, ...: not estimated
- Quadrupole coefficient optimized with two linear trends
- State-vector adjusted to best fit the tracking data
- ...

Parameter	Unit	Symbol	LAGEOS	LAGEOS II	LARES
Semi-major axis	km	a	12 270.00	12 162.08	7 820.31
Eccentricity	-	e	0.0044	0.0138	0.0012
Inclination	deg.	i	109.84	52.66	69.49
Radius	cm	R	30.0	30.0	18.2
Mass	kg	M	406.9	405.4	383.8
Area/Mass	m <sup>2</sup> /kg	A/M	6.94×10 <sup>-4</sup>	6.97×10 <sup>-4</sup>	2.69×10 <sup>-4</sup>

Table 2. Models currently used, within the LARASE research program, for the analysis of the orbit of the two LAGEOS and LARES satellites. The models are grouped in gravitational perturbations, non-gravitational perturbations and reference frames realizations.

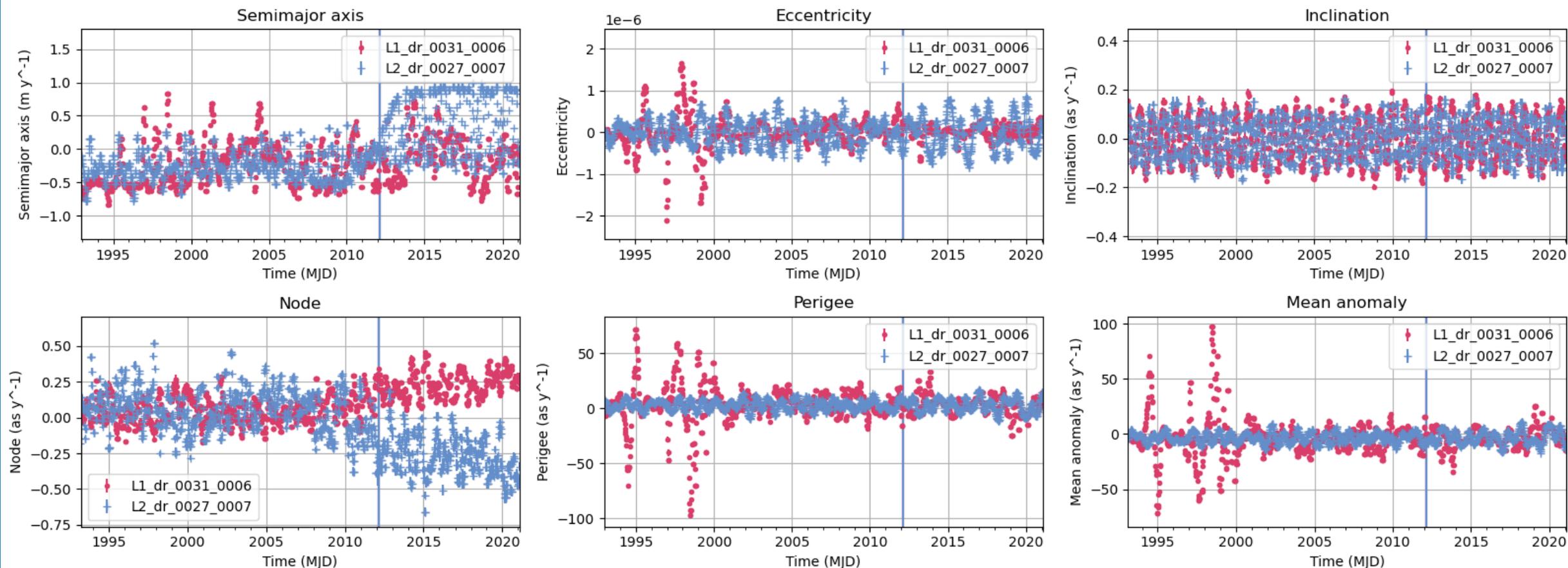
Model For	Model Type	Reference
Geopotential (static)	EIGEN-GRACE02S/GGM05S	[84,90,91]
Geopotential (time-varying, tides)	Ray GOT99.2	[92]
Geopotential (time-varying, non tidal)	IERS Conventions 2010	[89]
Third-body	JPL DE-403	[93]
Relativistic corrections	Parameterized post-Newtonian	[88,94]
Direct solar radiation pressure	Cannonball	[46]
Earth albedo	Knocke-Rubincam	[63]
Earth-Yarkovsky	Rubincam	[56,64,65]
Neutral drag	JR-71/MSIS-86	[50,51]
Spin	LASSOS	[42]
Stations position	ITRF2014	[95]
Ocean loading	Schernek and GOT99.2 tides	[46,92]
Earth Rotation Parameters	IERS EOP C04	[96]
Nutation	IAU 2000	[97]
Precession	IAU 2000	[98]



# POD and long-term behavior of LAGEOS II semi-major axis



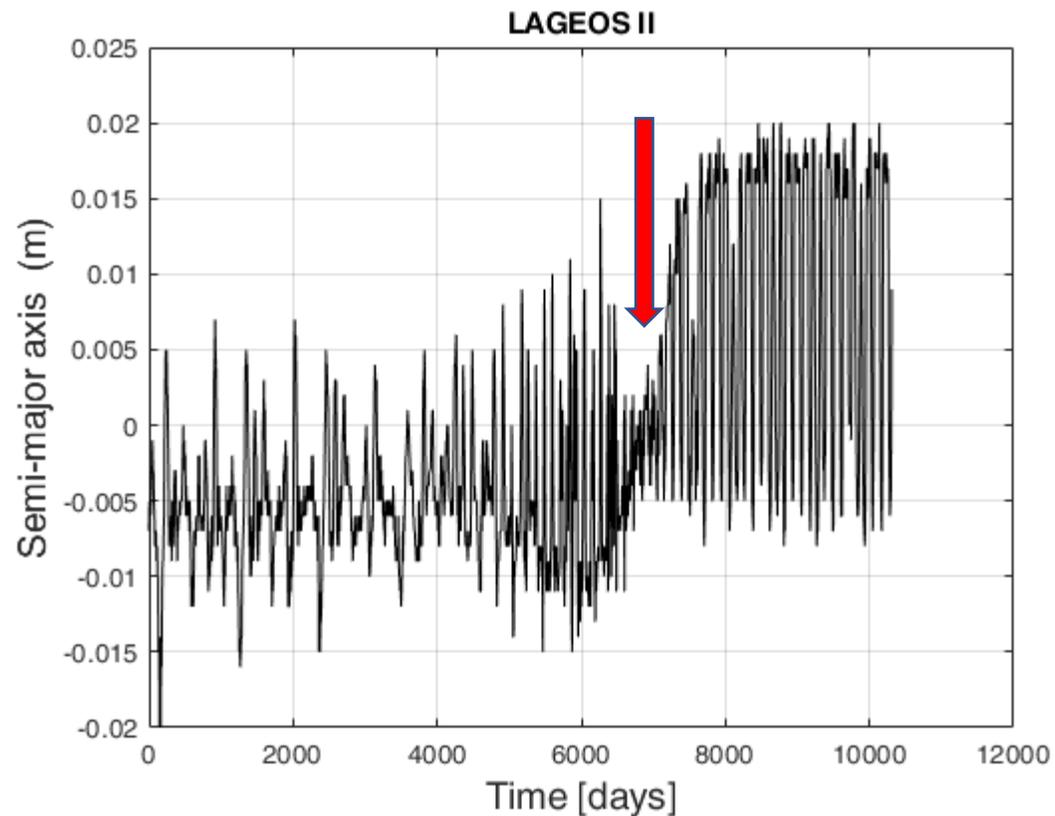
POD of **LAGEOS** and **LAGEOS II** on a time span of about 28 years: residuals in the rate of elements



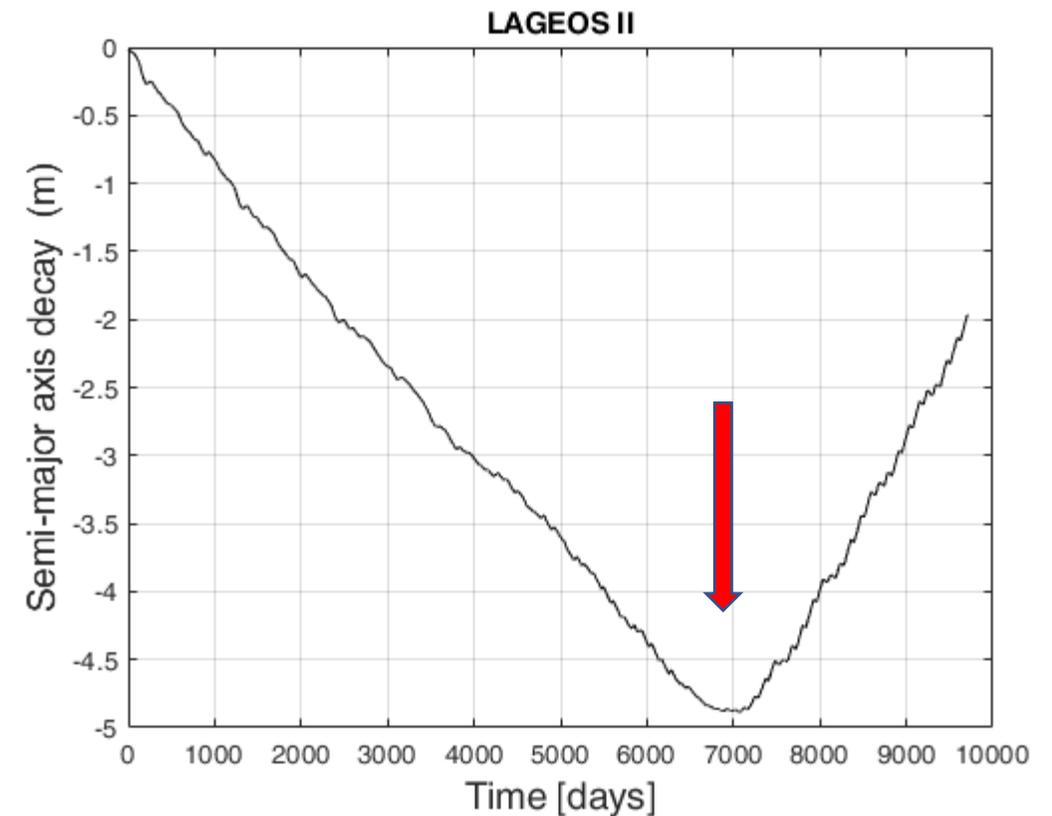
# POD and long-term behavior of LAGEOS II semi-major axis



Residuals in the semi-major axis (m/7d)



Integrated residuals in the semi-major axis (m)



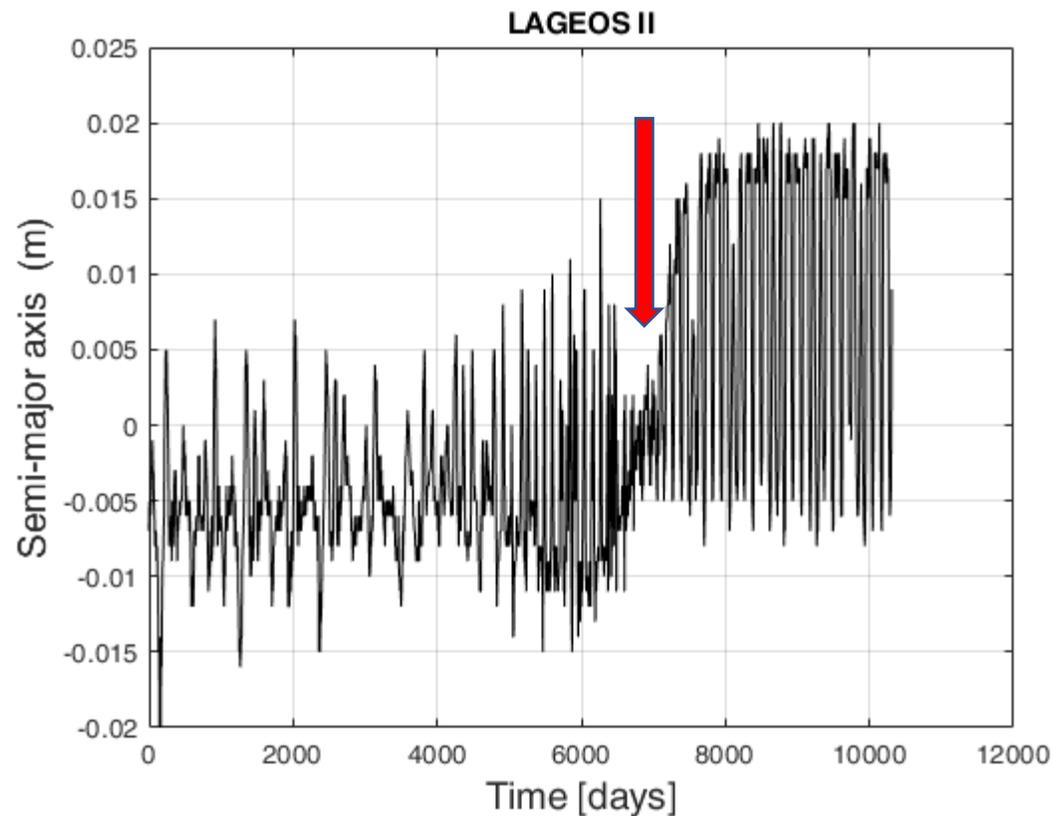
March 14, 2012

# POD and long-term behavior of LAGEOS II semi-major axis

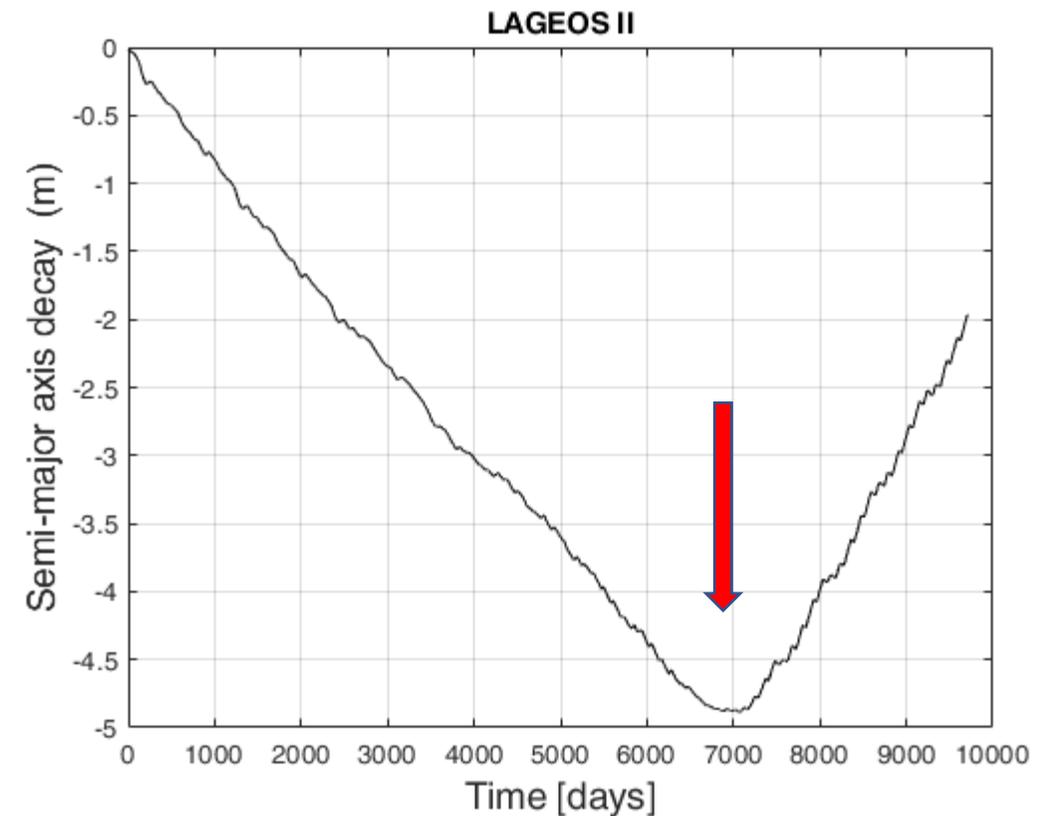


It is as if a certain mechanism is pumping energy to the satellite !

Residuals in the semi-major axis (m/7d)



Integrated residuals in the semi-major axis (m)



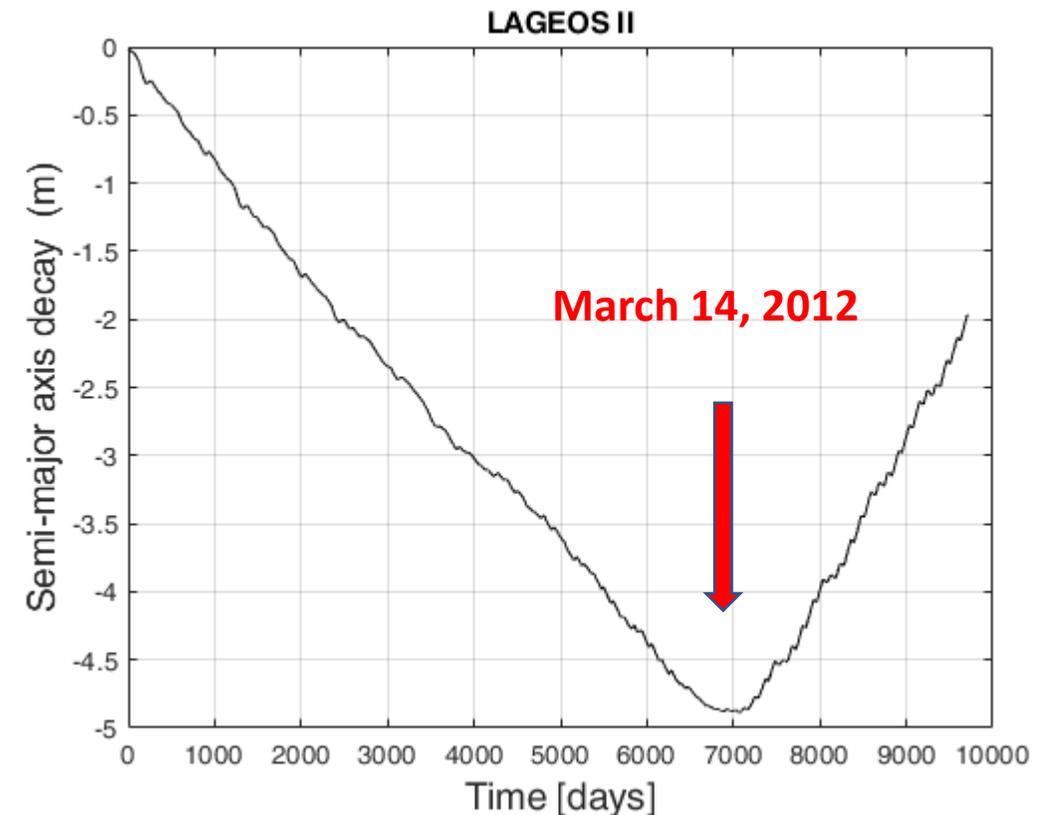
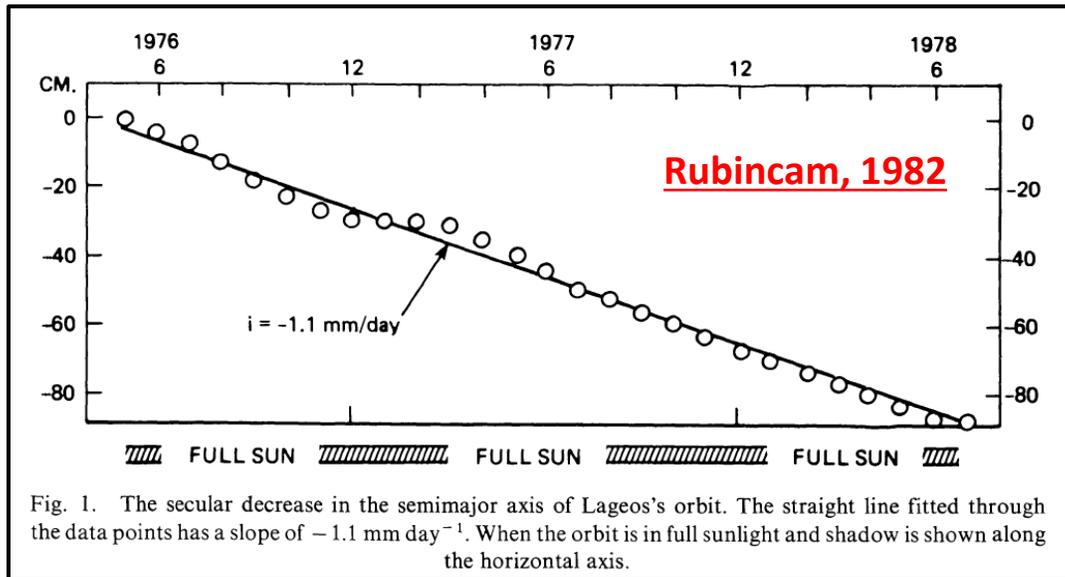
March 14, 2012

# POD and long-term behavior of LAGEOS II semi-major axis

## The former (old) explanation:

In the late 1980s and early 1990s, the observed decay for the semi-major axis of the two **LAGEOS** satellites was explained in terms of:

- Earth-Yarkovsky thermal drag  $\approx 70\%$
- Charged particles drag  $\approx 20\%$
- Neutral particles drag  $\approx 10\%$ .



# POD and long-term behavior of LAGEOS II semi-major axis

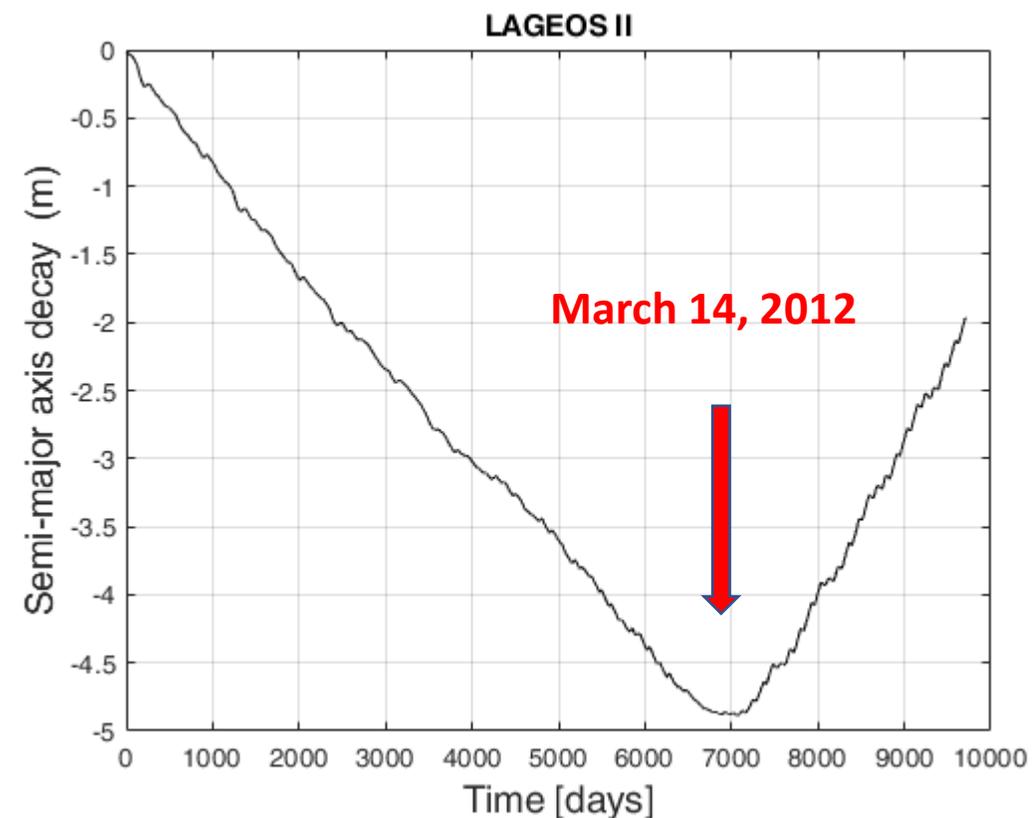
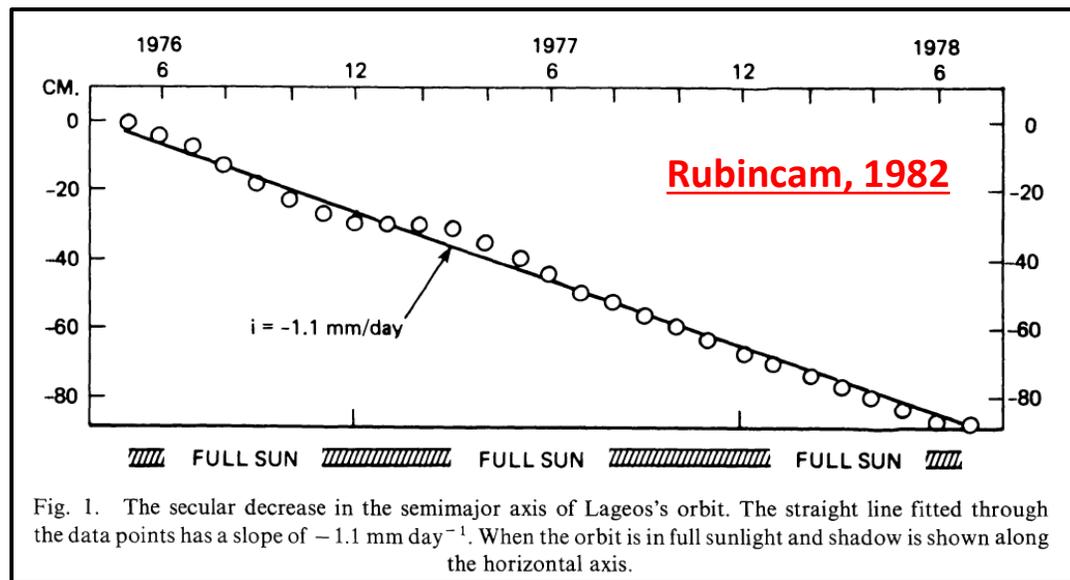


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Based on the results of our analyzes and the models we have developed for **NGPs**, we believe that the possible explanation for the observed phenomenon lies in the evolution of the **Spin** of **LAGEOS II** and its consequent impact on the **solar Yarkovsky** effect.



# Spin evolution of LAGEOS satellites



## Spin Models

The rotational dynamics of a satellite represents a very important issue that deeply impacts the goodness of the orbit modeling.

Indeed, the modeling of several disturbing effects (like the thermal thrust ones) depends on the knowledge of the spin period and orientation in the inertial space:

1. **Yarkovsky–Schach effect**
2. **Earth–Yarkovsky (Rubincam) effect**
3. **Asymmetric reflectivity from the satellite surface.**

Their modeling will greatly improve the **POD** of the two **LAGEOS** satellites avoiding the current (and significant) use of **empirical accelerations** during the data reduction.

A general theory is not easy to be developed because the modeling of the spin evolution depends on several factors:

- **spacecraft materials, its shape, details of the structure**
- **space environment: orbit altitude and inclination**
- **spin rate: kind of equations to be solved, resonances (rotational period, thermal inertia, orbit period).**

# Spin evolution of LAGEOS satellites



We have deeply reviewed previous spin models, in particular we:

- first built our own spin model in the rapid spin approximation
- adopted non-averaged torques in the equations to describe the slow spin approximation: we solved the problem of a metallic sphere rotating in an alternate magnetic field
- introduced in the equations all known possible torques (like in LOSSAM model)
- solved the equations in a body-fixed reference system in order to better describe the misalignment between the symmetry axis and the spin
- included in the equations the terms due to the transversal asymmetry
- carefully studied the satellites moments of inertia.

**M. Visco, D. Lucchesi**, *Review and critical analysis of mass and moments of inertia of the LAGEOS and LAGEOS II satellites for the LARASE program.* Adv. in Space Res. 57, 044034 doi:10.1016/j.asr.2016.02.006, 2016

**M. Visco, D. Lucchesi**, *Comprehensive model for the spin evolution of the LAGEOS and LARES satellites.* Phys. Rev. D 98, 044034 doi:10.1103/PhysRevD.98.044034, 2018

# Spin evolution of LAGEOS satellites



## The involved torques

We consider in the case of the two LAGEOS satellites four torques:

$$M_{mag}^E = V \sum_{i=1}^9 \frac{|B_i|^2}{2|\omega_s|} \{A_i'' [1 + \cos(2\omega_i t + 2\varphi_i)] - D_i' \sin(2\omega_i t + 2\varphi_i)\} \omega_s +$$

1. The magnetic torque (eddy currents)

$$V \sum_{i=1}^9 \frac{B_i \cdot \omega_s}{2|\omega_s|^2} \{[\alpha'(\omega_i) - A_i'] [1 + \cos(2\omega_i t + 2\varphi_i)] - [D_i'' + \alpha''(\omega_i)] \sin(2\omega_i t + 2\varphi_i)\} (\omega_s \times B_i) +$$

$$V \sum_{i=1}^9 \frac{B_i \cdot \omega_s}{2|\omega_s|} \{-A_i'' [1 + \cos(2\omega_i t + 2\varphi_i)] + D_i' \sin(2\omega_i t + 2\varphi_i)\} B_i$$

2. The gravitational torque

$$M_{grav}^b = 3\omega_{\oplus}^2 \{ \hat{s}^b \times [I_x (\hat{s}^b \cdot \hat{x}^b) \hat{x}^b + I_y (\hat{s}^b \cdot \hat{y}^b) \hat{y}^b + I_z (\hat{s}^b \cdot \hat{z}^b) \hat{z}^b] \}$$

3. The asymmetric reflectivity torque ( $C_R$  differences)

$$M_{ar}^b = \nu \frac{2}{3} \rho^3 \frac{\Phi}{c} \Delta\rho C_R (\hat{z}^b \times \hat{s}_{\odot}^b) |\hat{z}^b \times \hat{s}_{\odot}^b|$$

4. The CoM offset torque (with respect to the center of geometry).

$$M_{off}^b = \nu \pi \rho^2 \frac{\Phi}{c} C_R (\mathbf{h}^b \times \hat{s}_{\odot}^b)$$

$$\frac{d\vec{L}}{dt} = \mathbf{M}_{mag} + \mathbf{M}_{grav} + \mathbf{M}_{ar} + \mathbf{M}_{offset}$$

Angular momentum evolution

# Spin evolution of LAGEOS satellites

M. Visco, D. Lucchesi, *Comprehensive model for the spin evolution of the LAGEOS and LARES satellites. Phys. Rev. D 98, 044034 doi:10.1103/PhysRevD.98.044034, 2018*



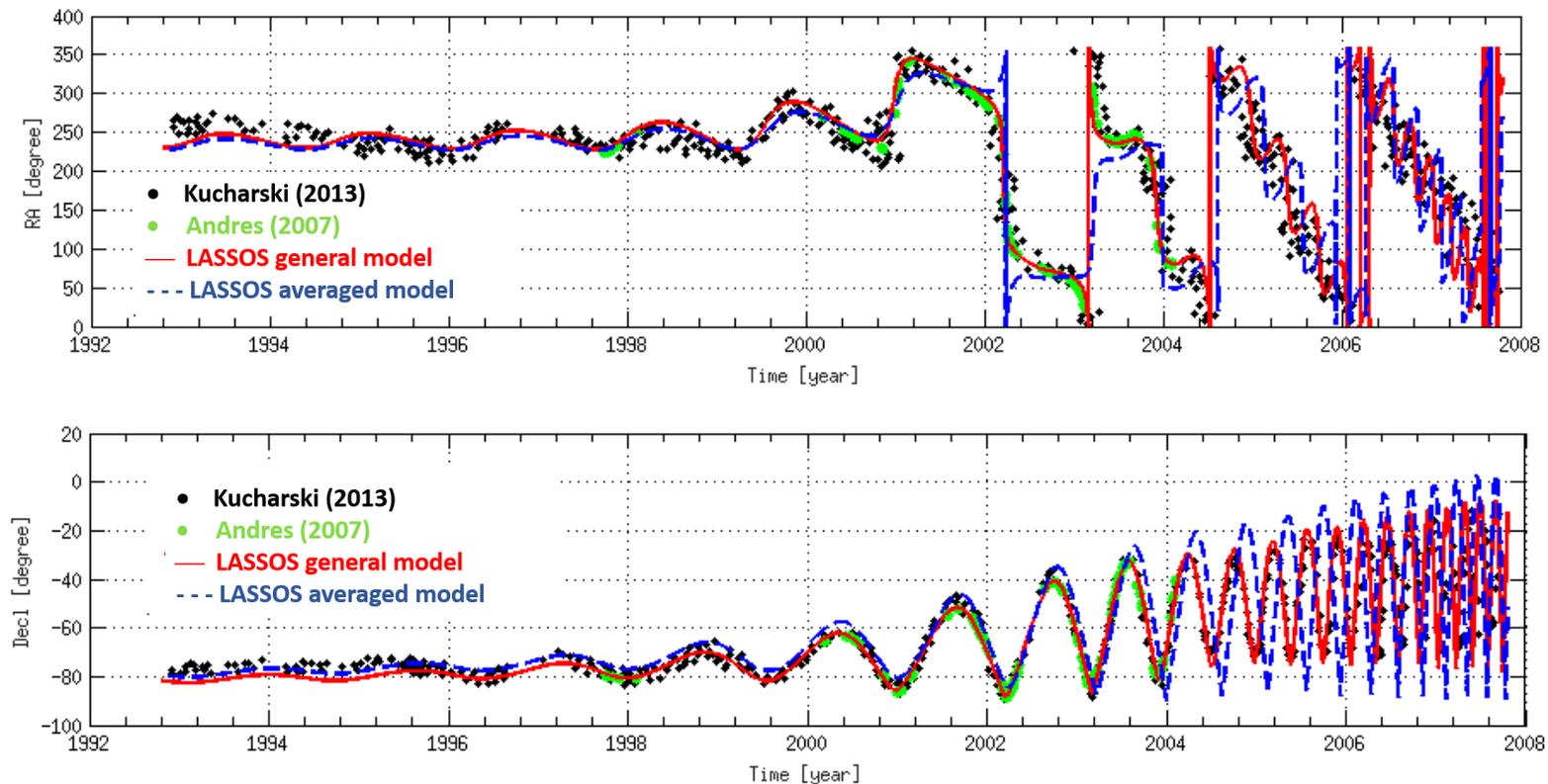
## LASSOS Spin Model: results for LAGEOS II

**Blue** = LASSOS model in the rapid-spin case  
**Red** = LASSOS model in the general case

## LArase Satellites Spin mOdel Solutions (LASSOS)

### Spin Orientation: $\alpha$ , $\delta$

Andrés de la Fuente, J.I., 2007. *Enhanced Modelling of LAGEOS Non-Gravitational Perturbations* (Ph.D. thesis). Delft University Press. Sieca Repro, Turbineweg 20, 2627 BP Delft, The Netherlands.  
Kucharski, D., Lim, H.C., Kirchner, G., Hwang, J.Y., 2013. *Spin parameters of LAGEOS-1 and LAGEOS-2 spectrally determined from Satellite Laser Ranging data*. *Adv. Space Res.* 52, 1332–1338.



# Spin evolution of LAGEOS satellites



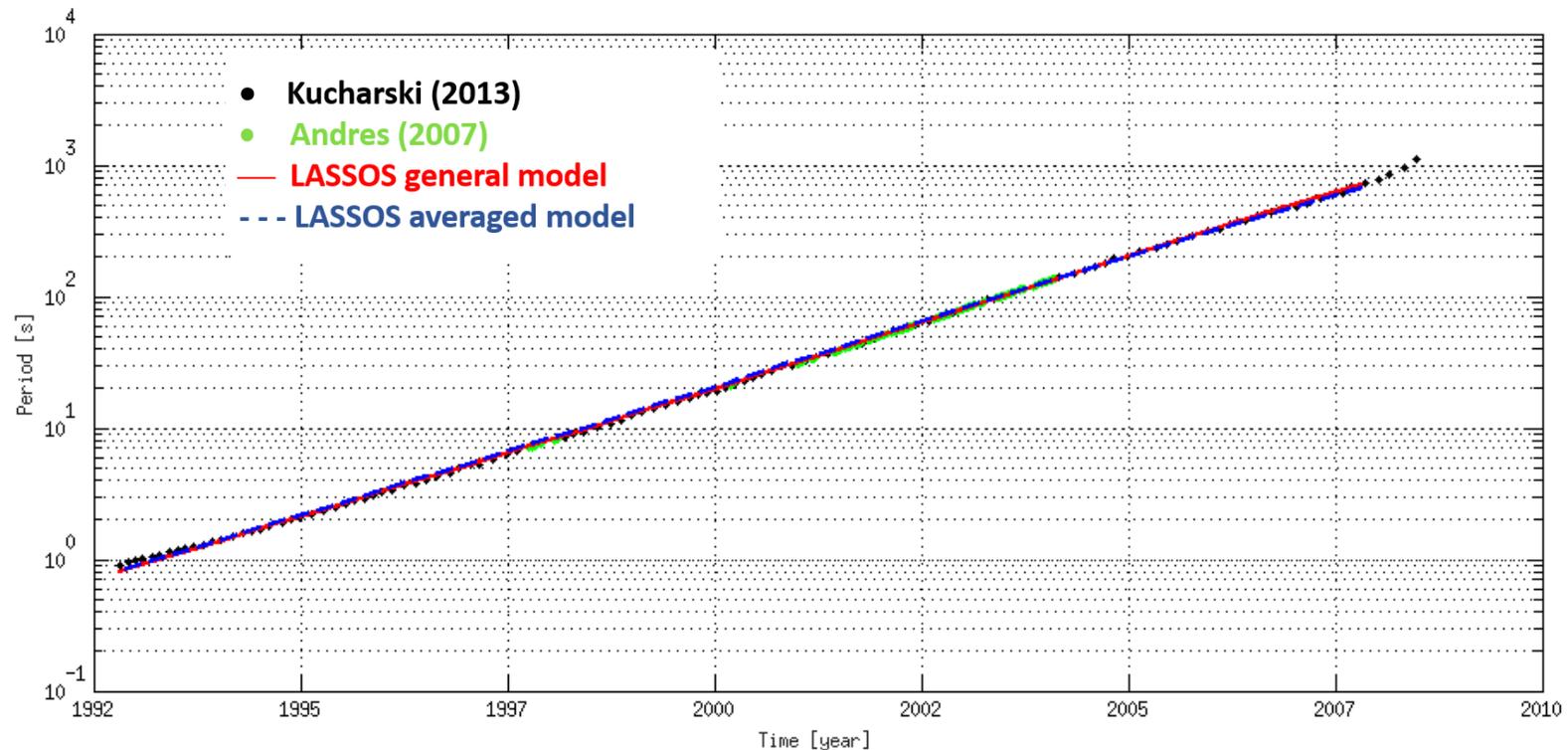
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## LArase Satellites Spin mOdel Solutions (LASSOS)

### Rotational Period: P



Andrés de la Fuente, J.I., 2007. *Enhanced Modelling of LAGEOS Non-Gravitational Perturbations* (Ph.D. thesis). Delft University Press. Sieca Repro, Turbineweg 20, 2627 BP Delft, The Netherlands.  
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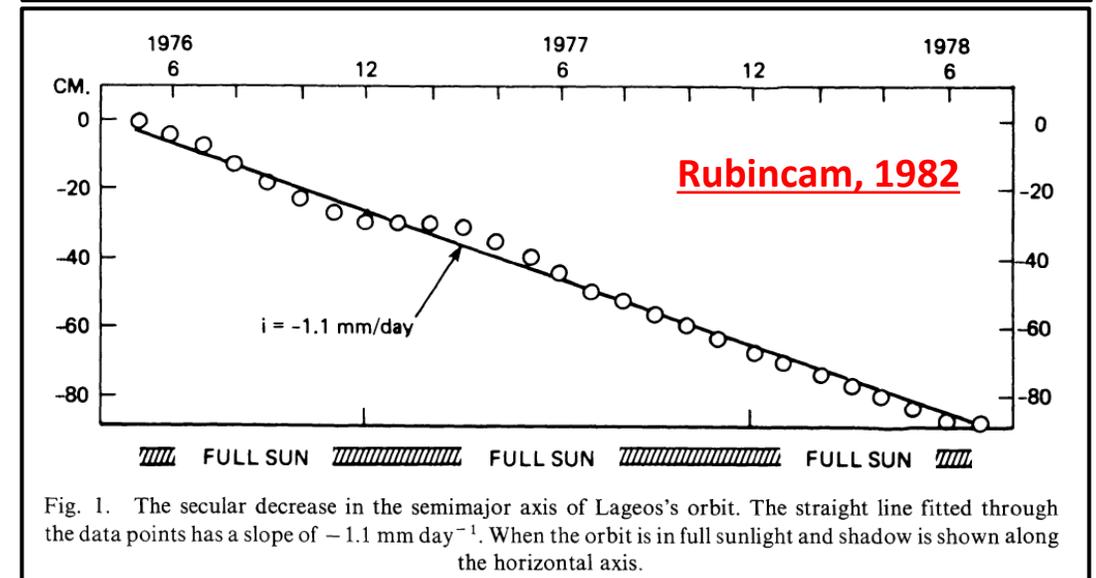
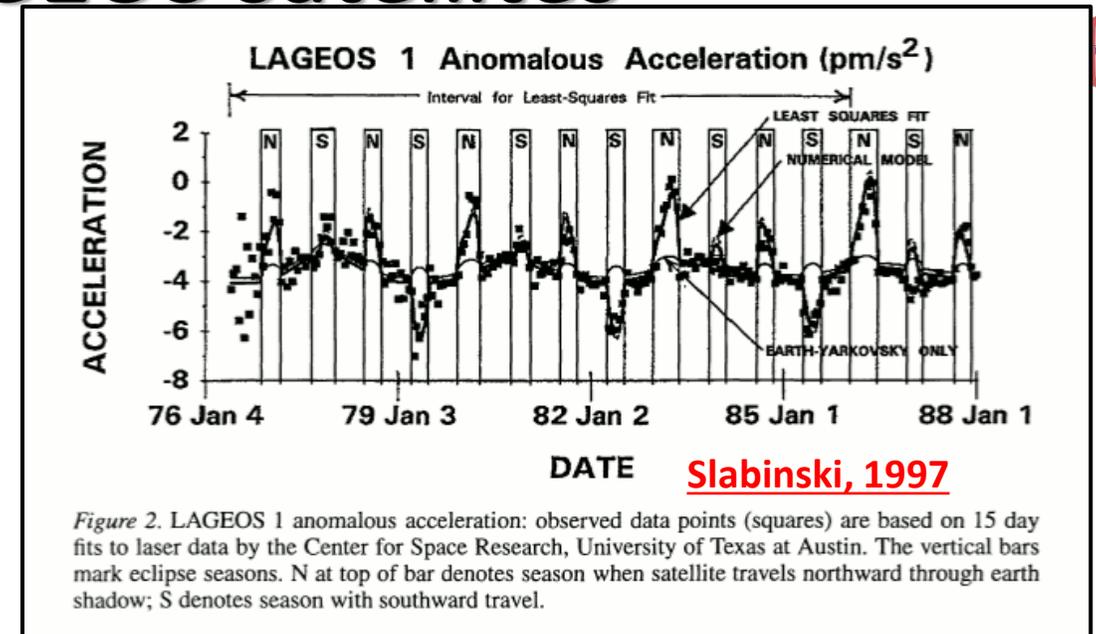
# Thermal thrust effects on LAGEOS satellites

An intricate role, among the complex **NGPs**, is played by the subtle **thermal thrust** effects that arise from the radiation emitted from the satellite surface as consequence of the non uniform distribution of its temperature.

In the literature of the older **LAGEOS** satellite this problem was attacked since the early 80s' of the past century to explain the (apparently) anomalous behavior of the **along-track acceleration** of the satellite, characterized by a complex pattern, and, consequently, of the satellite semi-major axis:

**Rubincam, Afonso, Ries, Scharroo, Farinella, Metris, Vokrouhlicky, Slabinski, Lucchesi, Andres, ...**

represents a non exhaustive list of the researchers that have successfully worked on this very important issue.

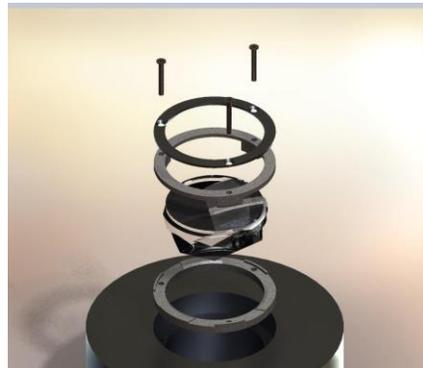
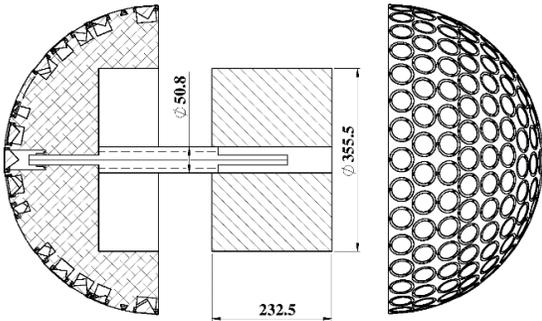


# Thermal thrust effects on LAGEOS satellites



The dynamical problem to solve is quite complex and should account for the following main aspects:

- A deep physical characterization of the satellite
  - **emission and absorption coefficients, thermal conductivity, heat capacity, thermal inertia, ...**
- Rotational dynamics of the satellite
  - **Spin orientation and rate.**
- Radiation sources
  - **Sun and Earth.**



We have tackled the problem following the two approaches considered in the past in the literature (but with some differences):

1. We developed a simplified thermal model of the satellite based on
  - **the energy balance equation on its surface**
  - **a linear approach for the distribution of the temperature with respect to its equilibrium (mean) temperature.**
2. A general thermal model based on
  - **a satellite (metallic structure) in thermal equilibrium**
  - **the CCRs rings are at the same temperature of the satellite**
  - **for each CCR the thermal exchange with the satellite is computed.**

# Thermal thrust effects on LAGEOS satellites



Simplified (averaged eqs.) thermal model: **Yarkovsky-Schach** effect

Characteristic amplitude:  $A_{YS} \cong \frac{16}{9} \frac{A}{m} \frac{\varepsilon \sigma}{c} T_0^3 \Delta T$

The model is similar to

Farinella P, Vokrouhlicky D., Thermal force effects on slowly rotating, spherical artificial satellites - I. Solar heating, *Plan. Space Sci.* 44, 12 (1996)

Accelerations in body:

With no eclipses

$$a_X = A_{YS} \frac{\sin z_{\odot}}{1 + (\omega_{spin} \tau)^2}$$

$$a_Y = A_{YS} \frac{\sin z_{\odot}}{1 + (\omega_{spin} \tau)^2} \omega_{spin} \tau$$

$$a_Z = A_{YS} \cos z_{\odot}$$

With eclipses

$$a_X = A_{YS} \frac{\sin z_{\odot}}{1 + (\omega_{spin} \tau)^2} \Gamma_X$$

$$a_Y = A_{YS} \frac{\sin z_{\odot}}{1 + (\omega_{spin} \tau)^2} \Gamma_Y$$

$$a_Z = A_{YS} \cos z_{\odot} \Gamma_Z$$

• We used:

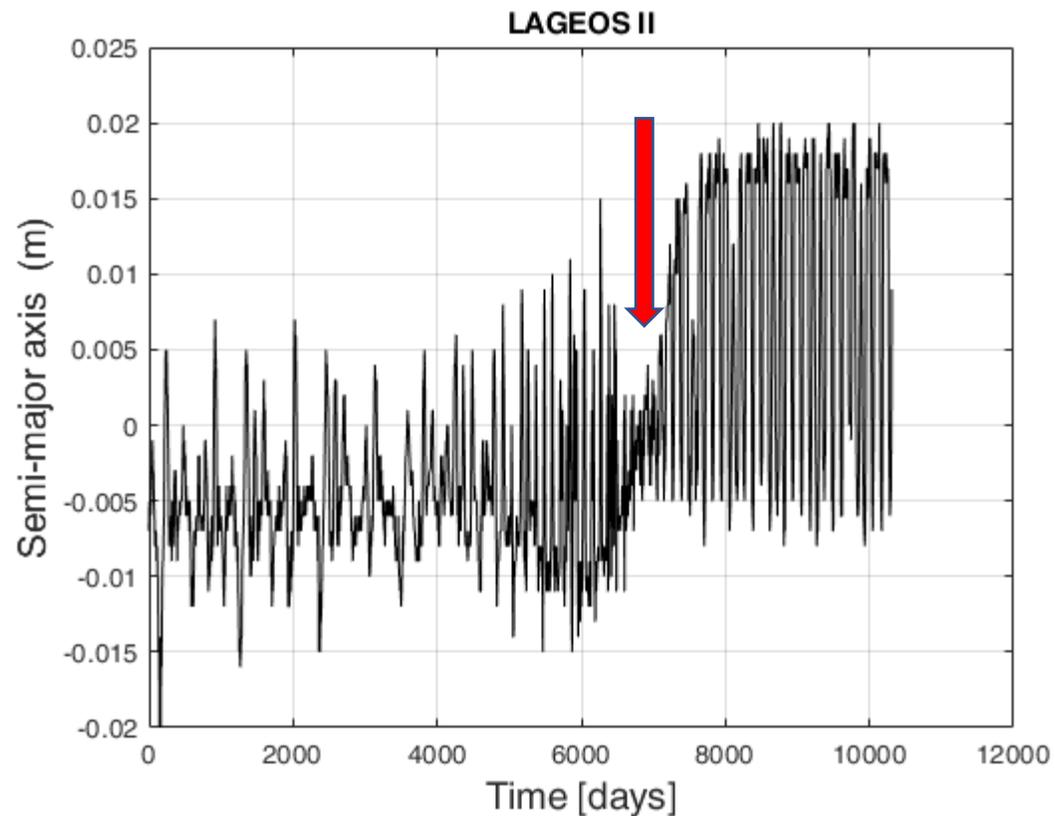
- $A_{YS} \cong -1.035 \times 10^{-10} \text{ m/s}^2$
- $\tau \cong 2113 \text{ s}$

Lucchesi D.M., Reassessment of the error modelling of the non-gravitational perturbations on LAGEOS II and their impact in the Lense-Thirring derivation - Part II, *Plan. Space Sci.* 50 (2002)

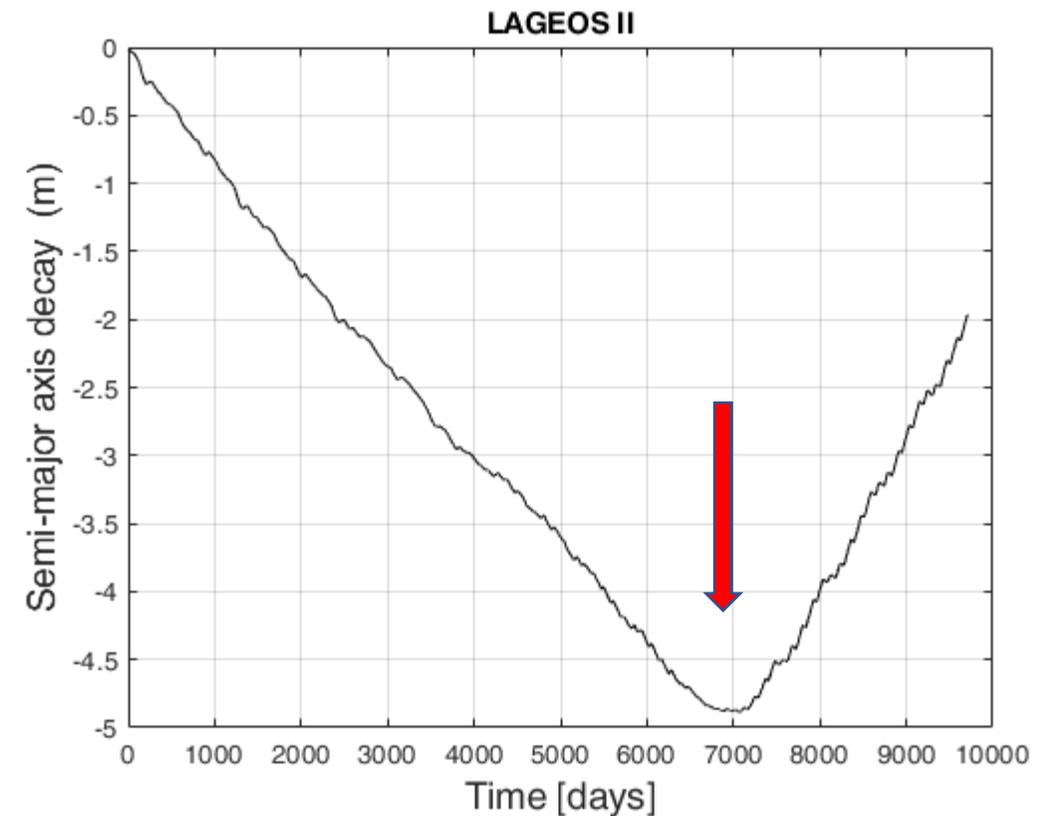
# On the decay and rise of LAGEOS II semi-major axis



Residuals in the semi-major axis (m/7d)



Integrated residuals in the semi-major axis (m)



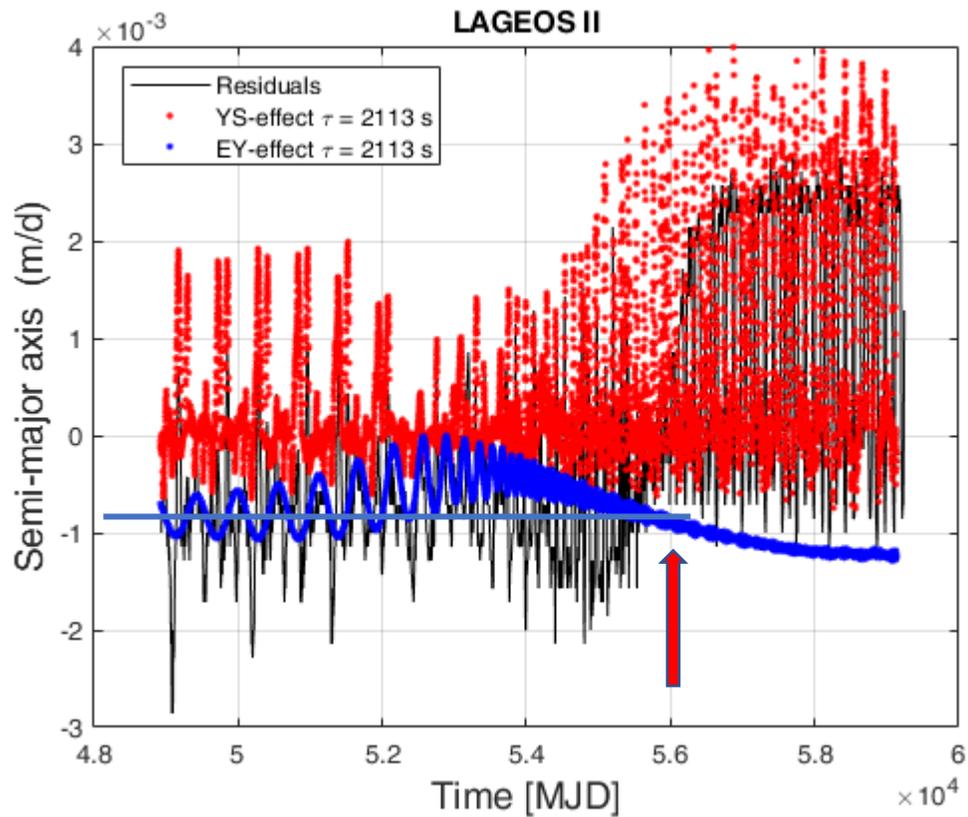
March 14, 2012

# On the decay and rise of LAGEOS II semi-major axis

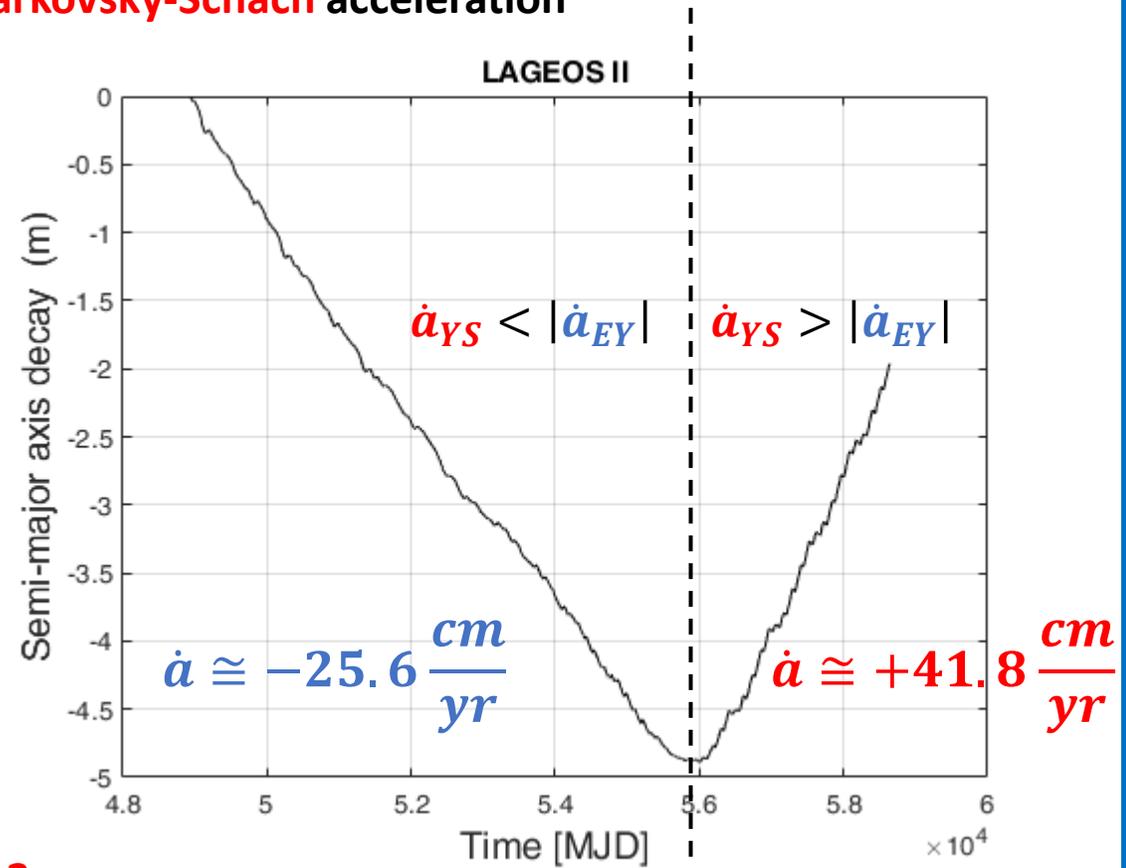


Residuals in the semi-major axis and their comparison with the solar Yarkovsky-Schach effect

Earth-Yarkovsky deceleration vs. solar Yarkovsky-Schach acceleration



$$\dot{a} \cong \frac{2}{n} T$$



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# On the decay and rise of LAGEOS II semi-major axis

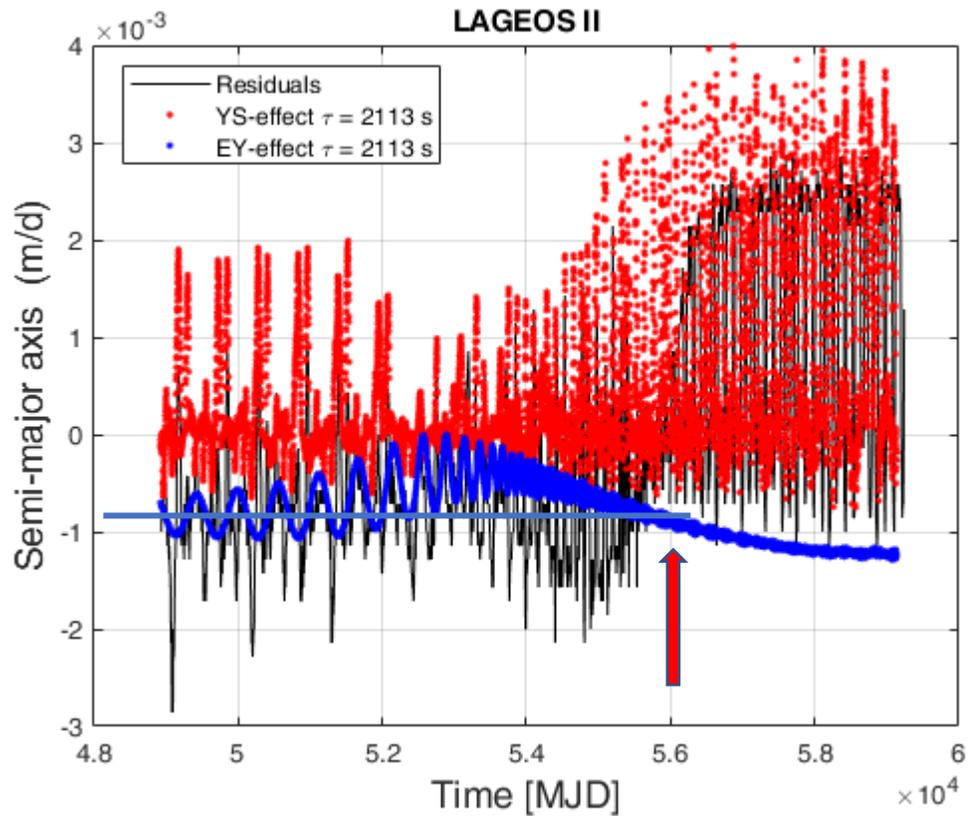


It seems evident that the change in the semi-major axis is due to the greater positive amplitudes assumed by the YS effect.

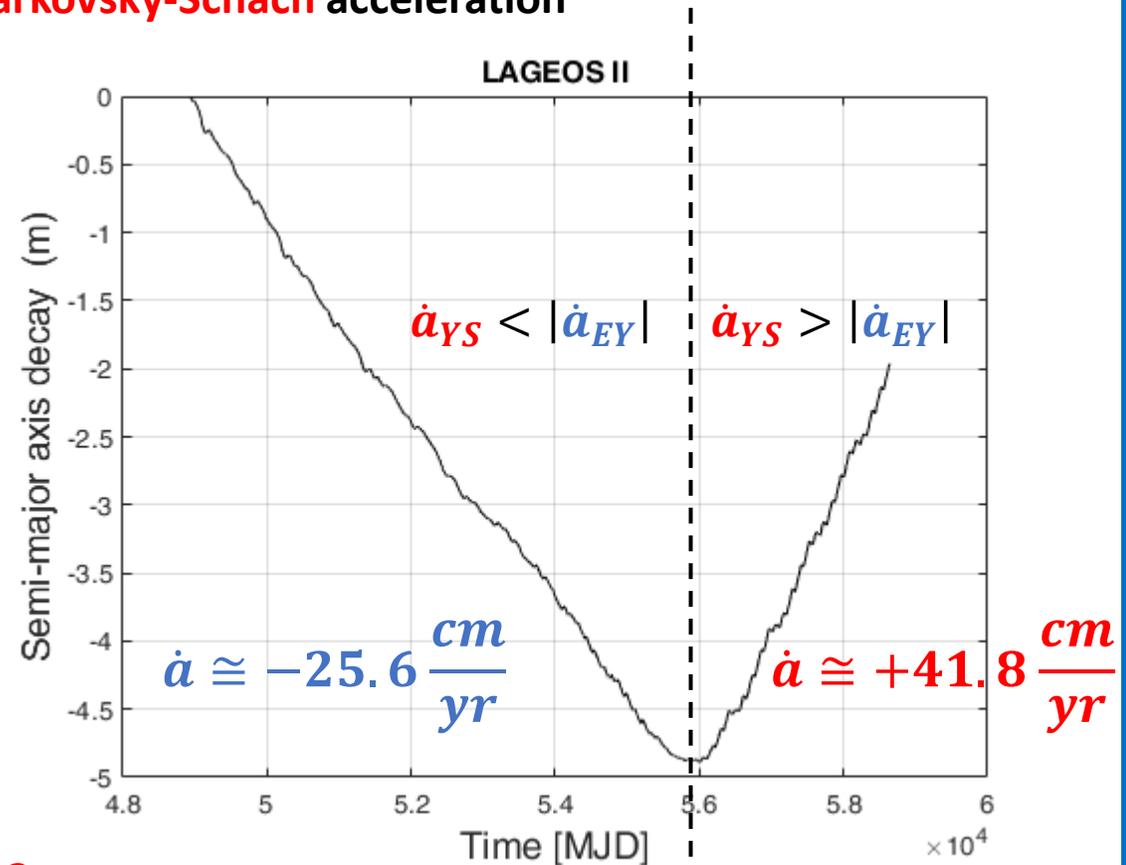
WHY?

Residuals in the semi-major axis and their comparison with the solar Yarkovsky-Schach effect

Earth-Yarkovsky deceleration vs. solar Yarkovsky-Schach acceleration



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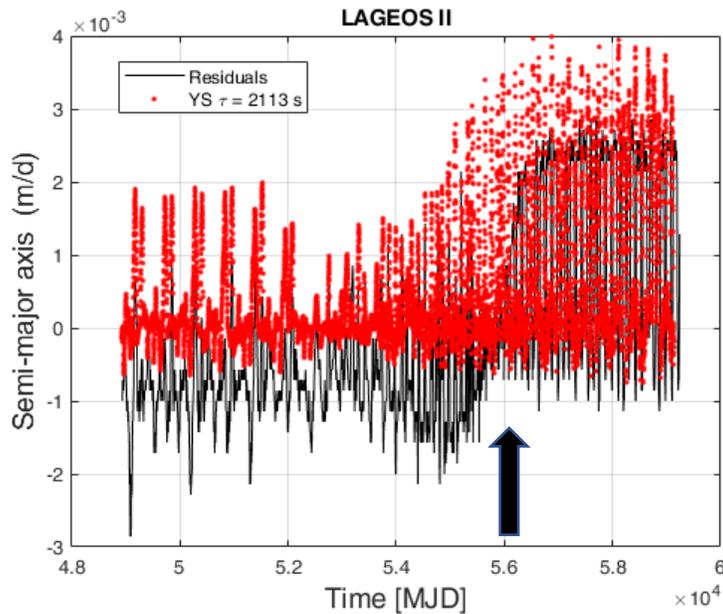
Residuals in the semi-major axis and their comparison with the solar Yarkovsky-Schach effect

## The role of the Thermal Inertia

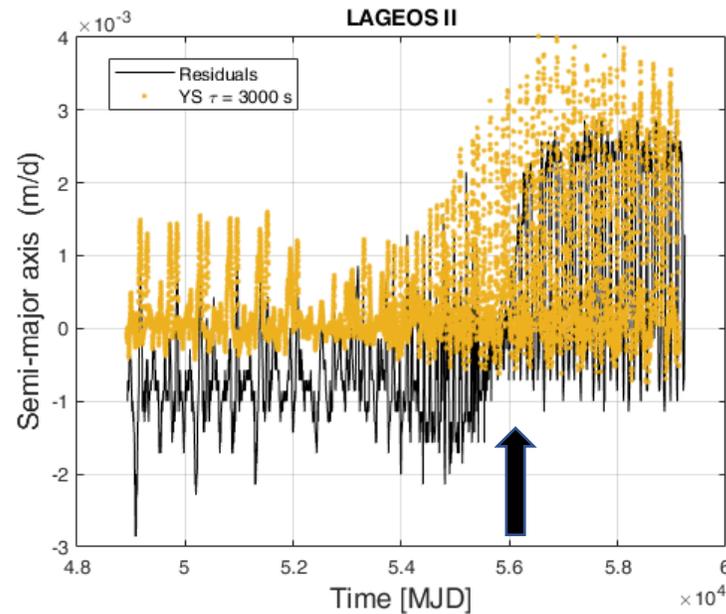
CCRs Thermal inertia:

- $\tau = 2113$  s
- $\tau = 3000$  s
- $\tau = 300$  s

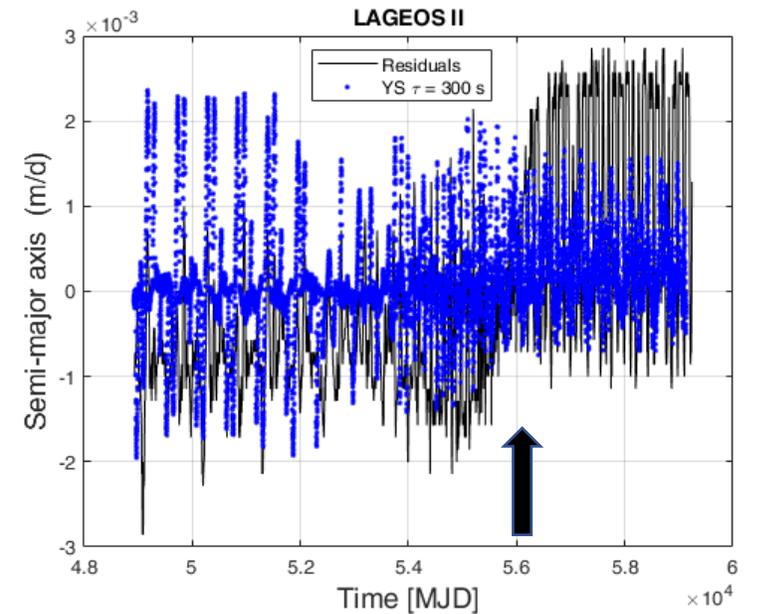
$$\frac{da}{dt} = \frac{2}{n\sqrt{1-e^2}} [T + e(T \cos f + R \sin f)]$$



$\tau = 2113$  s



$\tau = 3000$  s



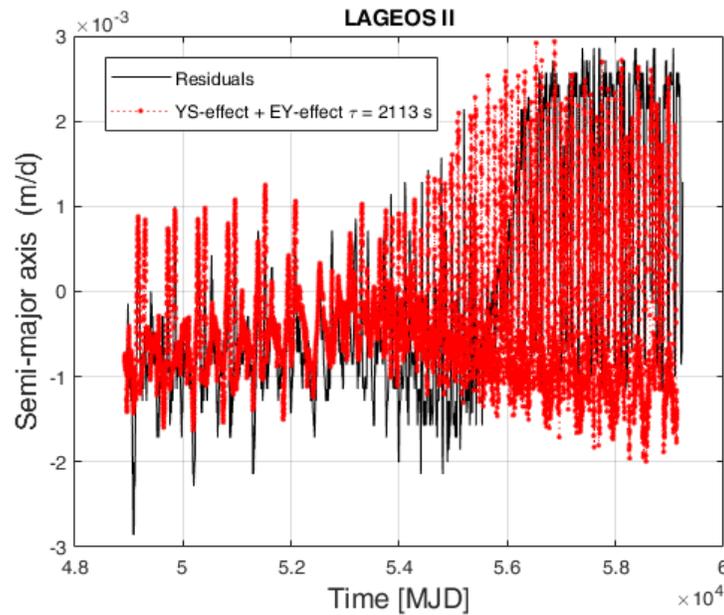
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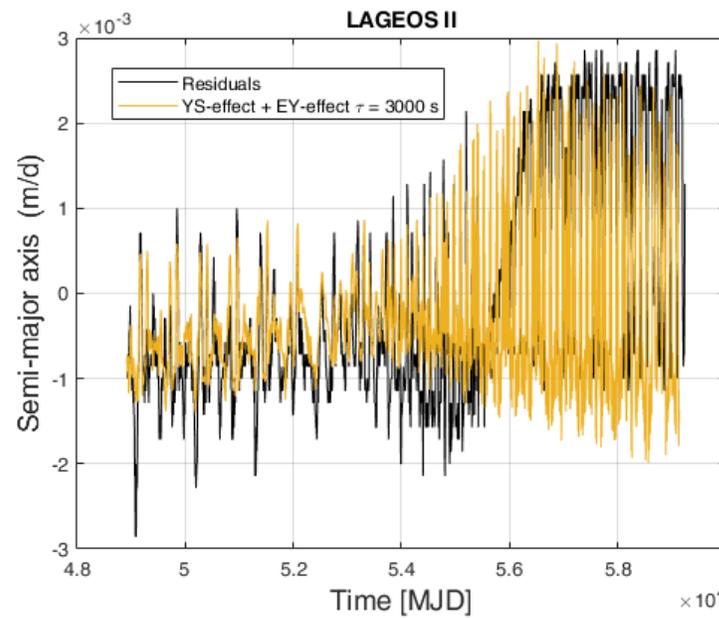


Residuals in the semi-major axis and their comparison with the solar Yarkovsky-Schach and Earth-Yarkovsky effects

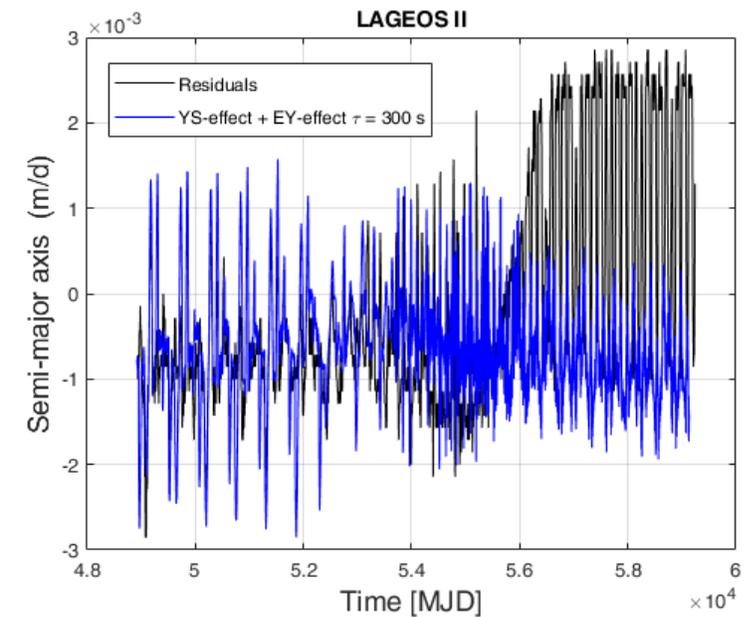
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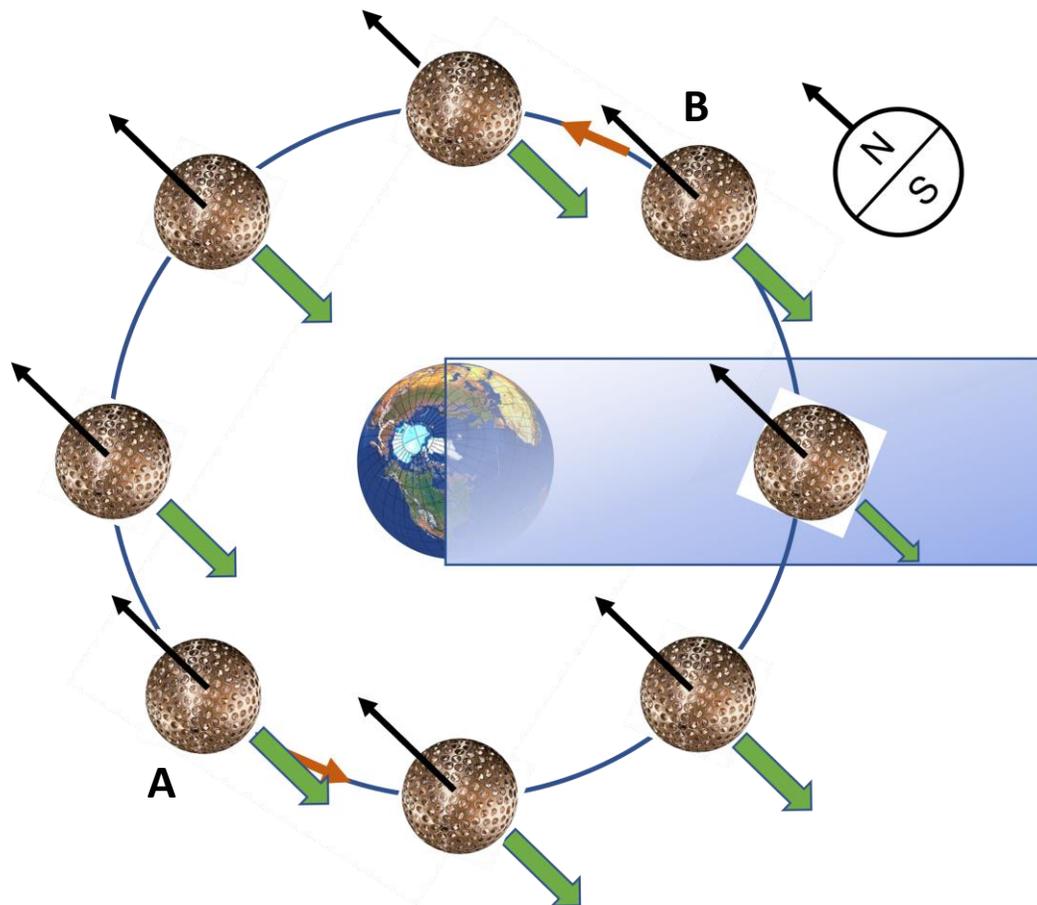


$\tau = 300$  s

# On the decay and rise of LAGEOS II semi-major axis



Spin in the orbital plane and direct motion of the satellite (**LAGEOS II**)



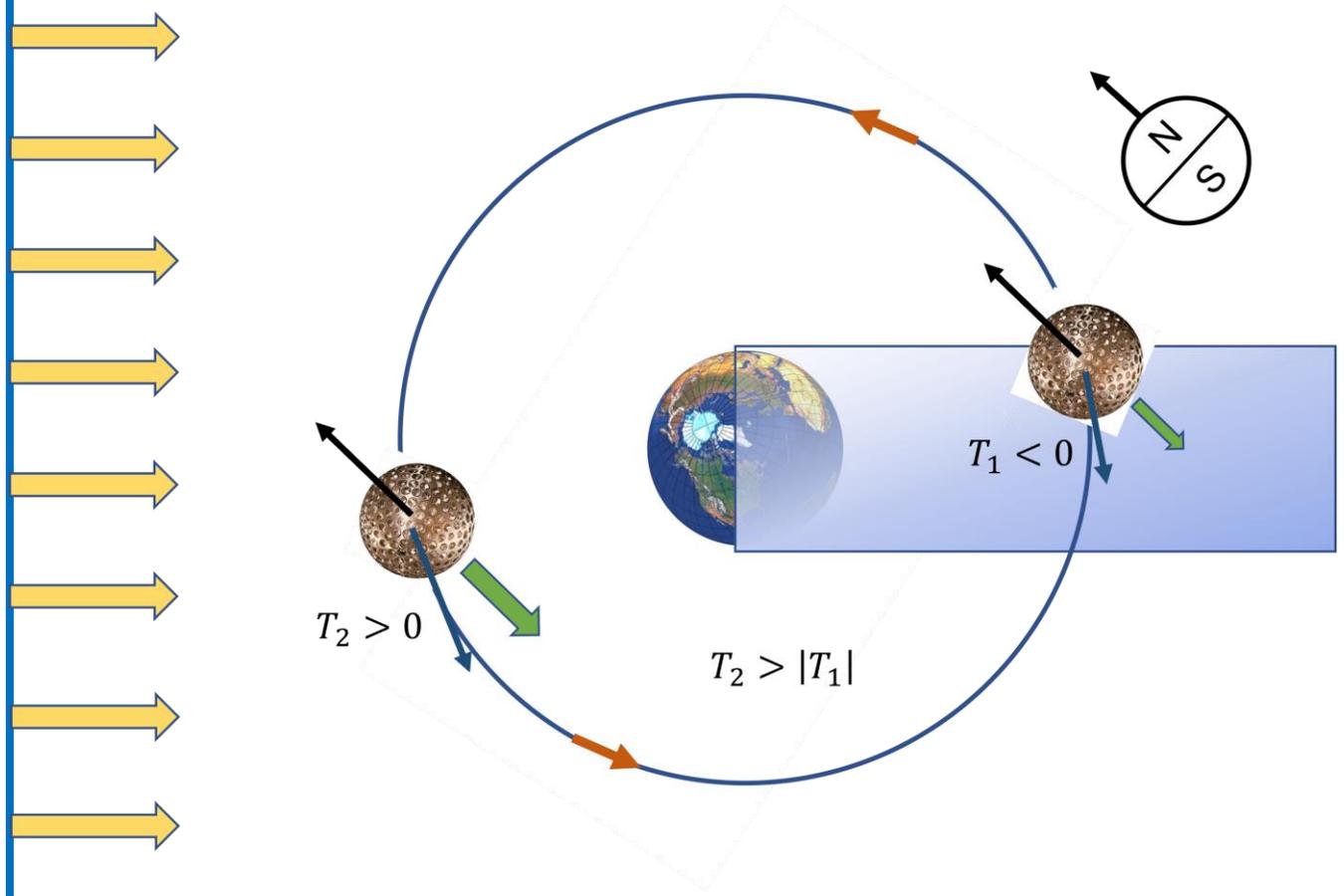
In this case we can talk about the seasonal Yarkovsky effect :

- In the absence of eclipses, the along-track component averages to zero and there are no long-term or secular effects on the semi-major axis
- In the presence of an eclipse, the force no longer averages to zero and long-term effects arise with a non-zero average
- The accelerations that are produced can be both positive and negative
- The sign depends on the relative geometry between the direction of the spin, the orbit and the position of the Sun
- The effect is greatest when the spin is in the orbital plane

# On the decay and rise of LAGEOS II semi-major axis



Spin in the orbital plane and direct motion of the satellite (LAGEOS II)



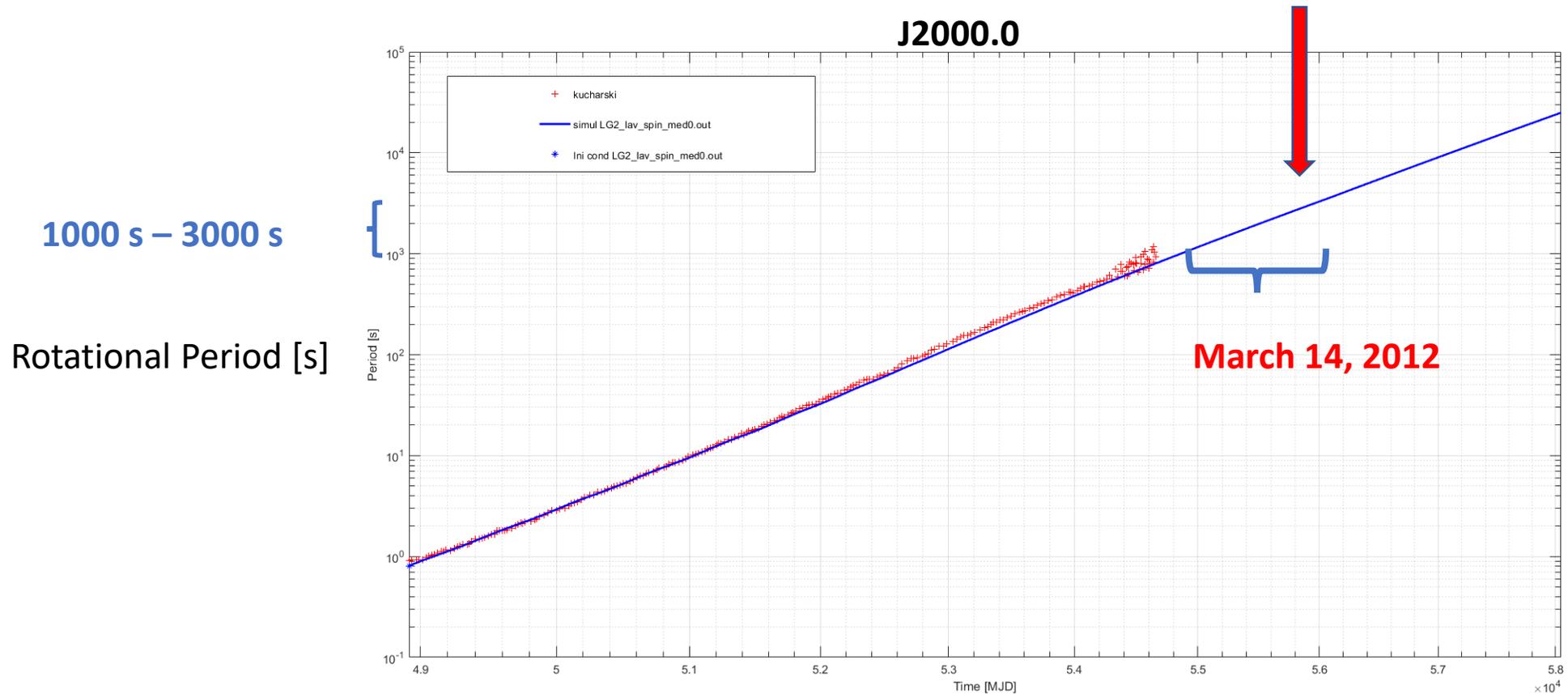
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- The accelerations that are produced can be both positive and negative
- The sign depends on the relative geometry between the direction of the spin, the orbit and the position of the Sun
- The effect is greatest when the spin is in the orbital plane
- In general, if the delay in re-emission due to thermal inertia is comparable with the duration of the eclipses, the positive accelerations are greater than the negative ones.

# On the decay and rise of LAGEOS II semi-major axis



The **LASSOS** (LArase Satellites Spin model Solutions) Spin model rotational period: **LAGEOS II**

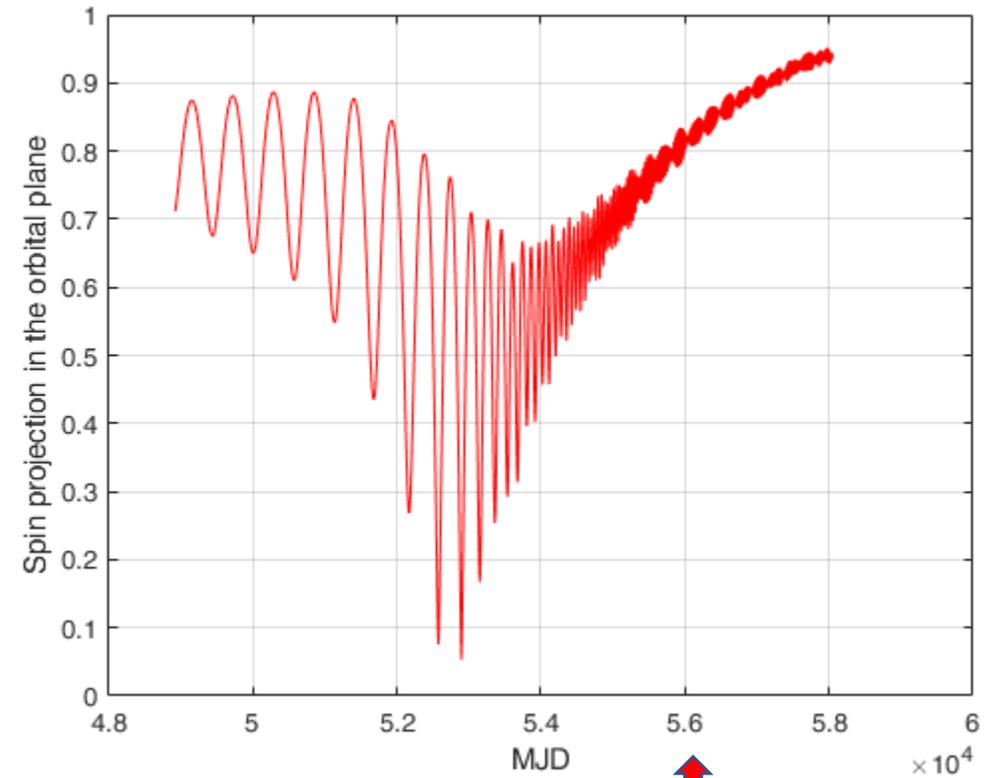
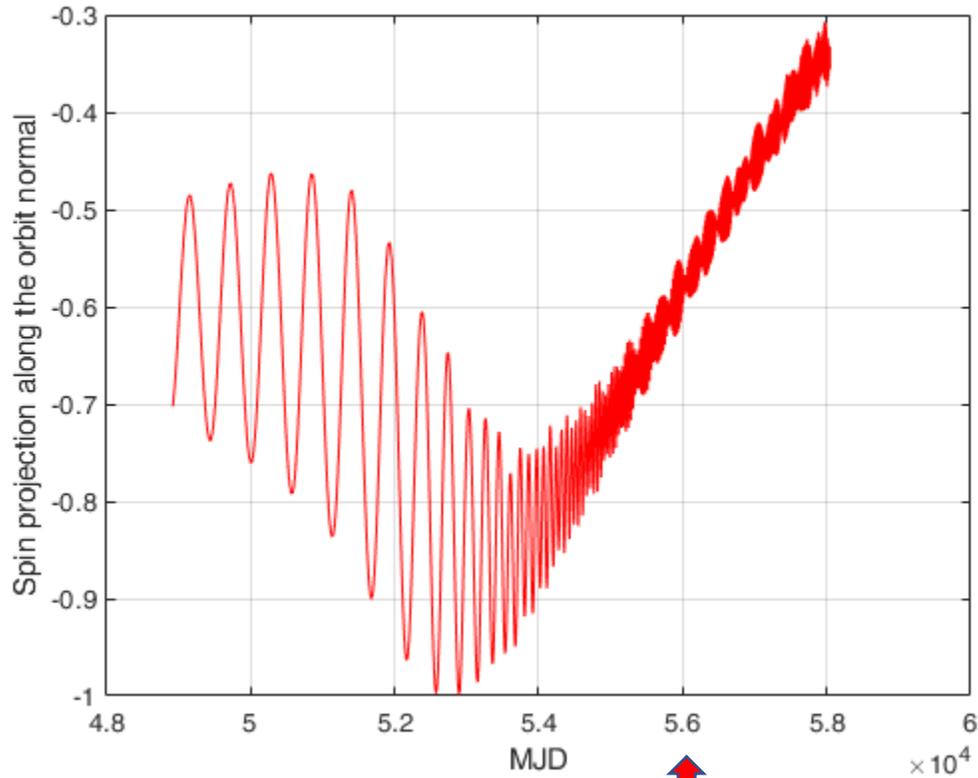


M. Visco, D. Lucchesi, *Comprehensive model for the spin evolution of the LAGEOS and LARES satellites*. Phys. Rev. D 98, 044034  
doi:10.3390/universe6090139, 2018

# On the decay and rise of LAGEOS II semi-major axis



LAGEOS II spin projections along the orbit normal and in the orbital plane using LASSOS



March 14, 2012

# On the decay and rise of LAGEOS II semi-major axis



Some open aspects to be explored:

- We have not yet modeled the entire **Earth-Yarkovsky** effect in the mediated model, but only its contribution from the perturbing acceleration acting along the satellite's axis of rotation: this represents the **main contribution** in the fast-spin approximation
- Other unmodeled **NGPs** (as the **asymmetric reflectivity**) should be considered for a more reliable comparison with the **orbital residuals**
- We are testing the goodness of the predictions of our general spin model (**LASSOS**) in the current slow-spin regime.

# Conclusions



- **Thermal thrust effects** are complex and subtle, and modeling them also requires knowledge of the evolution of the **satellite's spin**
- We have shown that the **decay and rise** of **LAGEOS II** semi-major axis can be explained in term of the combined action of **thermal thrust effects**:
  - **Earth-Yarkovsky** and solar **Yarkovsky-Schach**
- The **Yarkovsky-Schach** effect seems to be responsible for the **increase** in the semi-major axis of **LAGEOS II** as a direct consequence of the positioning of the **satellite's spin** in the orbital plane, in such a way as to **maximize** the **positive peaks** produced by this perturbing effect in the semi-major axis
- The role of the **drag** produced by **charged** and **neutral particles** in causing the decay of the semi-major axis (at least in the case of **LAGEOS II**) would seem considerably reduced compared to what was previously thought
- The **CCRs thermal inertia** seems to be in the interval **2000 s – 3000 s**
- In the present analysis the **mediated thermal model** has been used, it will be interesting to apply the **general model LATOS** once completed.



**Many thanks for your kind attention**